INTEGRATION METHOD OF MEASURING Q OF THE MICROWAVE RESONATORS

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Summary. A method of measurement of the loaded Q of microwave resonators is described, based on the relation between the integral of the resonance curve and the Q of the cavity considered. The block diagram of the measuring set and some results of the verification experiments are also included.

Introduction. Most frequently Q is obtained from the resonance curve width, by measuring the frequency difference between half-power points and by utilizing the relation of this quantity to Q. In this way it is possible to measure with an accuracy of 1 % though at the expense of using precise and valuable measuring instruments such as a microwave counter, a frequency and level stabilized signal generator, a precision power meter and/or a precision calibrated microwave attenuator.[1] This Technical Note presents another original method of measuring the loaded quality factor of microwave resonators based on the relation between the Q factor and the value of the integral of the resonance curve.

Method. The power $P(\omega)$ transmitted through a transmission cavity with matched generator and load connected to its input and output line, respectively, as a function of frequency is

$$P(\omega) = P_o\left[1 + Q_L^2(\omega/\omega_o - \omega_o/\omega)^2\right]^{-1} \qquad /1/$$

where $P_o = P(\omega_o)$ is the power transmitted at resonance frequency ω_o of a given mode and Q_L is the loaded quality factor of cavity. The integration of the Eq./1/ in the vicinity of the resonance yields:

$$I = \int_{\omega_1}^{\omega_2} P(\omega)\,d\omega = P_o\omega_o Q_L^{-1}\operatorname{arctg}(Q_L\omega_s\omega_o^{-1}) \qquad /2/$$

where $\omega_s = \omega_2 - \omega_1$ and $\omega_1 < \omega_o < \omega_2$.

When the values of the power transmitted at resonance, the cavity resonance frequency and the integration interval are known, then Eq./2/ may be used for calculating Q_L from the measured integrated value of the transmitted power-bandwidth product of the cavity considered. The block diagram of the measuring set is shown in Fig.1.

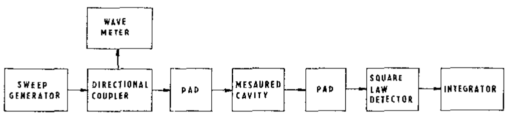

Fig.1. Block diagram of the measuring set.

It consists of a microwave sweep generator, pad attenuators, the resonator to be measured and a square law detector. The output signal from the detector is integrated in an integrator. As a matter of fact this integration is in the time domain and thus a more convenient formula for calculating Q_L is

$$I_t = P_o\omega_o k^{-1}Q_L^{-1}\operatorname{arctg}(Q_L\omega_s\omega_o^{-1}), \qquad /3/$$

where I_t is the value of integral obtained after integration over the time interval of the sweep from ω_1 to ω_2 and $k = \omega/t$ is the sweep rate. In order to preserve the validity of the quasi-static formulation of the Eq./3/, the sweep rate is considered to be sufficiently low. The transcendental equation /3/ is solvable only graphicaly or by the iterative method. However in the many cases when the sweep range is many times greater than the bandwidth of the measured resonator, the calculation of Q_L can be simplified as follows: When the integration interval is infinite, then the value of the integral of the resonance curve is $I_{t\infty} = \lim_{\omega_s\to\infty} I_t = 0.5\pi P_o\omega_o k^{-1}Q_L^{-1}$. From this equation it is possible to calculate approximative value of the quality factor Q_L' if instead of $I_{t\infty}$ the measured value of I_t is substituted: $Q_L' = 0.5\pi P_o\omega_o k^{-1}I_t^{-1}$. The relative difference ε_Q between approximative and correct values of Q 's is $\varepsilon_Q = (Q_L' - Q_L)/Q_L = \operatorname{arccotg}(\omega_S Q_L\omega_o^{-1})/\operatorname{arctg}(\omega_S Q_L\omega_o^{-1})$, /e.g. for $\omega_S Q_L\omega_o^{-1} > 7$ it is $\varepsilon_Q < 0.1$ /. The calculated approximative value of the quality factor Q_L' can be substituted for Q_L in the term arctg(.) of the Eq./3/ thus reducing Eq./3/ to a linear equation. Owing to the shape of the function arctg(.) for higher values of argument, the error is negligible. The resulting formula for the corrected cavity quality factor Q_{LK} is finally

$$Q_{LK} = P_o\omega_o k^{-1}I_t^{-1}\operatorname{arctg}(0.5\,\omega_S P_o k^{-1}I_t^{-1}) \qquad /4/$$

The value of power P_o transmitted at resonance may also be measured by means of the integrator when the time of integration is unity and the generator is tuned to the fixed frequency ω_o. Since quantities P_o and I_t appear in Eq./4/ as a proportion, the integrator need not be absolutely calibrated in the units W.Hz of the transmitted power-bandwidth product.

Results. The suggested method was verified with some X band cavities, whose loaded quality factor was previously measured by means of a bandwidth method of accuracy 2 %. The results shown in the Table 1 represent the mean of the series of ten measurements. Q_{LK} or Q_L are Q 's of the cavities obtained either by integration or the bandwidth method, respectively.

Table 1.

Cavity No	Q_{LK}	Q_L	$(Q_{LK} - Q_L)/Q_L$
0111	7 538 ± 231	7 820 ± 156	-3.6×10^{-2}
0155	11 745 ± 159	11 200 ± 220	4.9×10^{-2}

The results obtained by the integration method differed at most by about 5 % from those obtained by the reference method. The major measuring error sources of the integration method are the deviation from the square law of the detector, the level instability of sweep generator, the nonlinearity of the integrator and the drift of the integrator, especially when the sweep rate is very low. The advantages of the method include its simplicity and the unsophisticated measuring equipment. The proposed method does not demand any precision measurement of the half-power frequencies as is currently encountered in the bandwidth method when applied to the difficult case of the high Q cavities. Naturally, the integration method likewise the bandwidth method are not applicable to the distributed resonant circuits.

References.

1. M. Sucher, "Measurement of Q," in Handbook of Microwave Measurements, vol. 2, M. Sucher and J. Fox, Ed. Brooklyn, NY: Polytechnic Press, 1963, ch. VIII, pp. 417-493.

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